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SIMPLIFIED PREDICTION OF BALANCE SENSITIVITY

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ABSTRACT

It is possible to estimate the first and second critical speeds of a rotor by making use of measurements during a coast-down.

Combining the knowledge of the critical speeds with the geometry and weight of the rotor, it is possible to estimate the sensitivity to static and dynamic unbalance.

The rotor should be operating in the "rigid rotor" region as opposed to the "flexible rotor" region.

BIOGRAPHICAL SKETCH

Brian C. Howes, M.Sc., P. Eng., is the Chief Engineer at Beta Machinery Analysis Ltd. in Calgary, Alberta, and has been involved in machinery troubleshooting for both rotating and reciprocating equipment since 1972. He is one of the foremost experts in vibration analysis.

Scott Masterson is also a Project Engineer with Beta Machinery Analysis Ltd.

INTRODUCTION

This paper describes a technique for balancing machines at speeds between their first and second shaft criticals in a single run.

When a machine has no balancing history, conventional balancing procedures will often require two or three trial weight runs to determine sensitivities and to balance the rotor. Unfortunately, costly down-time often prevents the completion of such a job.

The method of STATIC – DYNAMIC BALANCING uses the following assumptions to simplify the problem:

1. The rotor and its supports are approximately symmetrical about the centre of mass.

2. Two plane balancing is being performed.

3. Static and dynamic components of vibration are being considered as opposed to the "raw" vibration.

4. The machine approximates a rigid rotor/flexible bearing system. (Refer to Figure 1)

The experimental technique requires the collection of "as found" and "coast-down" vibration data. These data, along with some of the physical characteristics of the machine, will enable a calculation of both the static and dynamic balance sensitivities of the system. Some sample calculations will follow.

The reasoning behind this type of balancing is that a two plane balancing problem can be effectively reduced to two single plane problems that can be solved simultaneously in one run.

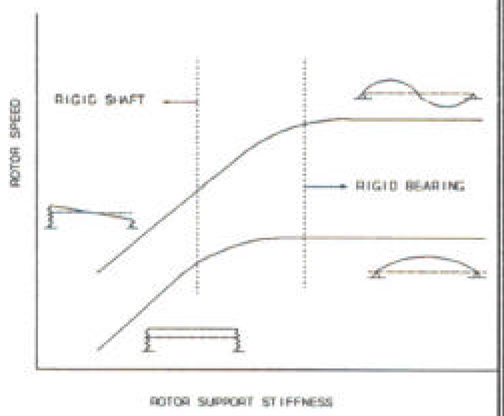

FIGURE 1 Rotor Speed vs. Support Stiffness

PROCEDURE

Static – Dynamic balancing proceeds as follows:

1. Collect "As Found" vibration data.
    - Proximity probes give the best results for rigid pedestals.
    - Shaft absolute (Shaft stick) data is more appropriate when pedestal vibration is significant.
2. Obtain displacement amplitude and phase data from the coast-down.
3. After performing runout subtraction, resolve the "as found" and coast-down data into static and dynamic components. (Refer to: Rotor Balancing by Static-couple Derivation, John M. Csokmay).
4. Construct Bode or Nyquist plots of both the static and dynamic components of the rundown.
5. Determine the first (static) critical from coast-down data, and ensure that the operating speeds is below the second (dynamic) critical.
6. Estimate the total rotating mass.
7. Calculate the theoretical static and dynamic balance sensitivities (See calculations that follow procedure.)
8. Calculate both the static and dynamic masses required at each balance plane from the sensitivities.
9. Determine the angles by which the high spots lag the heavy spots from the two coast down plots. Locate the light spot for both static and dynamic unbalances on each balance plane.
10. Vectorially sum the masses required at each balance plane and attach the two resultant masses at the required locations.

CALCULATING STATIC AND DYNAMIC BALANCE SENSITIVITIES

This method operates under the premise that both static and dynamic (couple) unbalances are independent and have separate balance sensitivities. These sensitivities are calculated as follows:

Static Sensitivity

The force from static unbalance mass, m_s, rotating at a radius, r, and speed, w

Equals:

The force of the entire rotating mass, M rotating about an eccentric distance, e_s at speed, w.

Or:

$(m_s)(r)(w)^2 \;=\; (M)(e_s)(w)^2$

The static sensitivity can be given in the following terms:

$S_s \;=\; (m_s)(r)(e_s) \;=\; M$

Static Sensitivity: (continued)

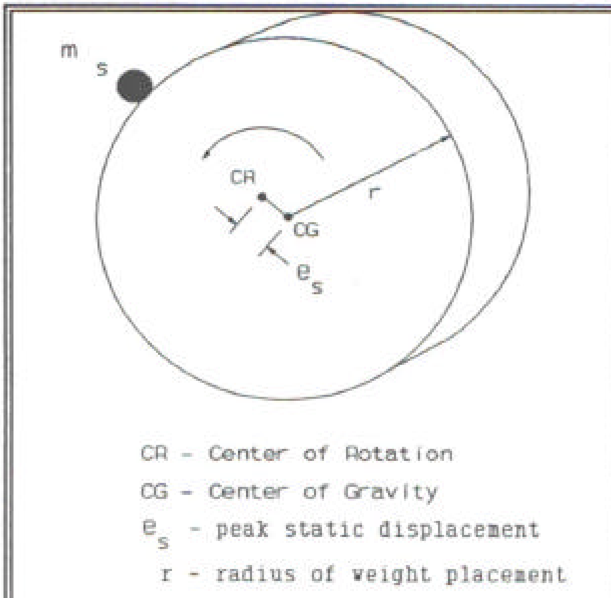

FIGURE 2 Static Unbalance

Dynamic Sensitivity: (See Figure 3)

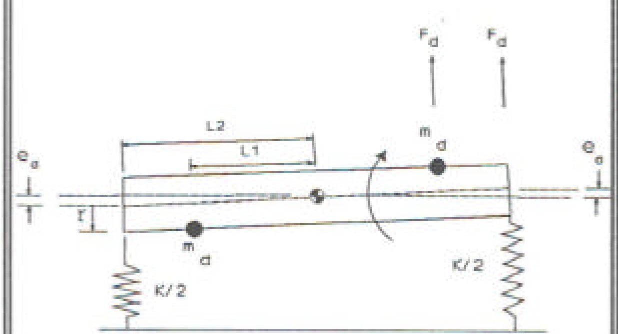

FIGURE 3 Dynamic Unbalance

The force from the dynamic unbalance mass, m_d rotating at a radius, r and speed, w

Equals:

The dynamic force opposing the stiffness, K at each bearing.

The total bearing stiffness is calculated as follows:

$W_c = (K/M)^{1/2}$

$K = M w_c^2$

But stiffness at a bearing is K/2

The dynamic force at the bearing opposing this stiffness is:

$F_d = K/2\,(e_d) = 0.5\,M(e_d)(w_c)^2$

The force from the dynamic unbalance mass at each balance plane is:

$F_d = (m_d)(r)w^2$

But the dynamic forces at the bearings are reduced by the ratio of L_1/L_2:

$F_d = F_d'\,(L_1/L_2)$

Let: $c = (L_1/L_2)$

$F_d = c\,(m_d)(r)w^2$

Equating Forces:
$c\,(m_d)(r)w^2 = 0.5\,M\,(e_d)w_c^2$

The dynamic sensitivity is then:

$$S_d = (m_d)(r)/(e_d) = \frac{M w_c^2}{2 c w^2}$$

Where:

M = total rotating mass
e_s = peak static component of displacement at bearing
e_d = peak dynamic component of displacement at bearing
m_s = total static mass
m_d = dynamic mass at each balance plane
r = radius of balance weights
w = rotational speed at which es is measured
w_c = first critical speed
S_s = static balance sensitivity
S_d = dynamic balance sensitivity
L_1 = distance from balance plane to centre of Mass
L_2 = distance from bearing to centre of mass
c = ratio of L_1/L_2
F_d = dynamic force at each bearing
F_d' = dynamic force at each balance plane
K = combined bearing stiffnesses

EXAMPLE 1:

SAMPLE BALANCE CALCULATIONS FOR A 30 MW GENERATOR

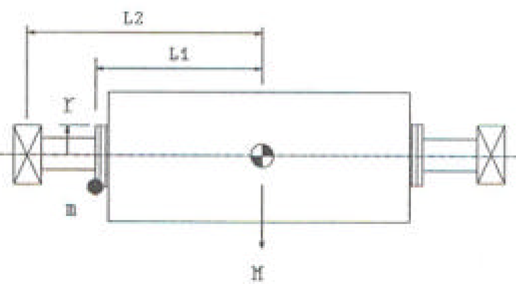

FIGURE 4 Generator Rotor

Total rotor mass: M = 11,800 kg
Operating Speed: w = 3600 rpm
1st Critical Speed: w_c = 1350 rpm
Radius for Balance Weights: r = 14"
L_1/L_2 ratio: c = 0.67

Estimate of Balance Sensitivities:

Static Sensitivity:

$S_s = (m_s)(r)/(e_s) = M$
S_s = 11,800 kg in/in pk
= 416 oz-in/mil pk
= <u>208 oz-in/mil p-p</u>

This is the total sensitivity.

104 oz-in/mil p-p will be required at each balance plane.

(manufacturers rated static sensitivity is 200 oz-in/mil p-p total)

Dynamic Sensitivity:

$$S_d = (m_d)(r)/(e_d) = \frac{M w_c^2}{2 c w^2}$$

$$S_d = \frac{(11{,}800 \text{ kg})(1350)^2}{2(.67)(3600)^2}$$

S_d = 1240 kg in/in pk
= 43.7 oz-in/mil pk
= <u>21.8 oz-in/mil p-p</u>

This is the sensitivity at each bearing. 21.8 oz-in/mil p-p will be required at each balance plane.

(manufacturers rated dynamic sensitivity is 14-21 oz-in/mil p-p)

Figures 5 and 6 demonstrate the static and dynamic components of the generator cost-down vibration. Data have been taken with proximity probes, and slow roll corrected. The angles by which high spots lag the heavy spots can be found from the total phase changes over the coast-down.

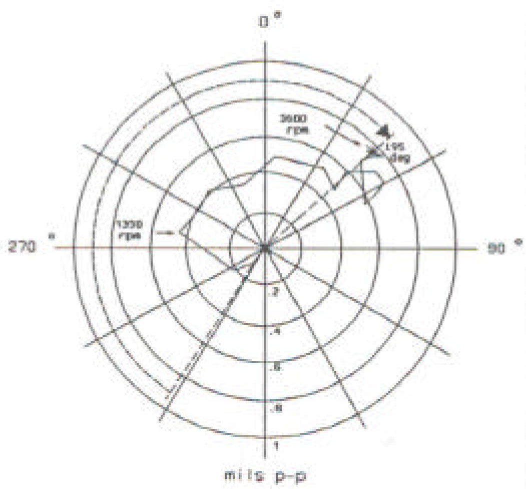

FIGURE 5 Static Component of Generator Coast-Down

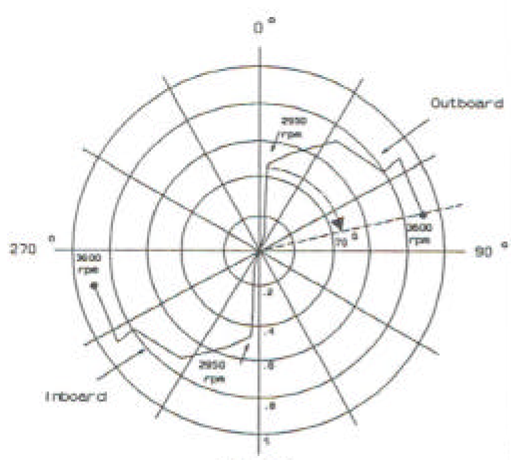

FIGURE 6 Dynamic Component of Generator Coast-Down

EXAMPLE 2: COAL DRYER EXHAUST FAN

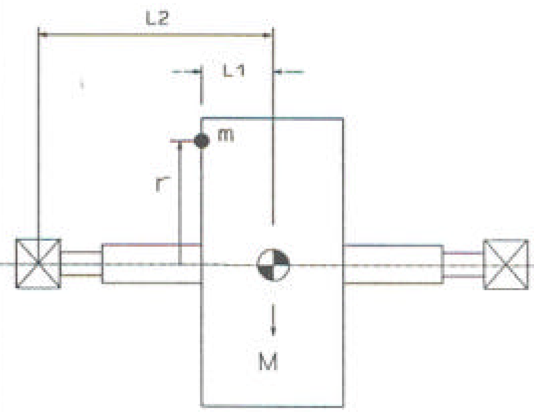

FIGURE 7 Dryer Exhaust Fan Rotor

Machine Layout

Total Rotor Mass	M	=	10,200 Kg
Operating Speed	w	=	1190 RPM
First Critical Speed	w_c	=	880 RPM
Radius for Balance Wts.	R	=	48”
L_1/L_2 Ratio	c	=	.22

Estimate of Balance Sensitivities

Using the same calculations as Example 1, the following sensitivities are obtained:

Estimated
Static Sensitivity: S_s = 180 oz-in/mil p-p

Estimated
Dynamic Sensitivity: S_D = 224 oz-in/mil p-p

Estimate of Balance Weight Placements

Figures 8 and 9 show static and dynamic rundown plots. The static lag angle (high spot lags heavy spot) is about 130° and the dynamic lag angle is about 75°.

The static and dynamic weights should be placed opposite the respective heavy spots.

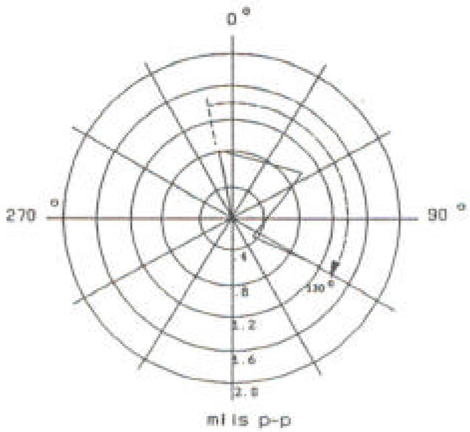

FIGURE 8 Static Component of Fan Coast-Down

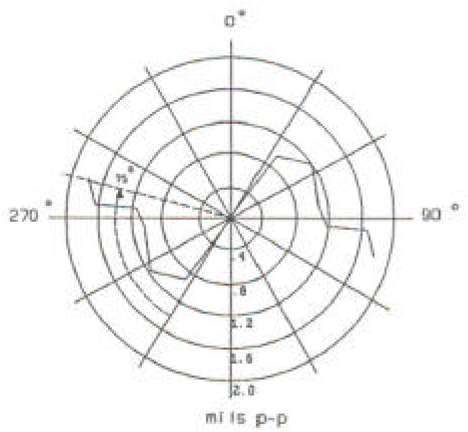

FIGURE 9 Dynamic Component of Fan Coast-Down

Actual Balance Sensitivities

A. As Found Case Vibrations:

Fan Outboard 1.91 mils p-p, 118°
Fan Inboard .80 mils p-p, 227°

Resolved As Found Case Vibrations:

Static Component = 0.90 mils p-p, 143°
Outboard Dynamic Component = 1.14 mils p-p, 99°
Inboard Dynamic Component = 1.14 mils p-p, 279°

B. Attach Balance Weights

These balance weights were determined from previously known overall sensitivities before the rotor mass was known.

Total Static Weight = 101 g
Dynamic Weight = 123 g per side

Vibration after placement of balance weights:
Static Component = .38 mils p-p, 179°
Outboard Dynamic Component = .30 mils p-p, 348°
Inboard Dynamic Component = .30 mils p-p, 168°

C. Weight Effect Vectors, T.

ie: T = Effect of Weights =
(Trial weight vibration) – (As found vibration)

T_{Static} = .38, 179° - .90, 143°
= 0.63 mils p-p, 302°

$T_{Dynamic\ Outboard}$ = .30, 348° - 1.14. 99°

= 1.28 mils p-p, 292°

Calculation of Actual Sensitivities:

Static Sensitivity, S_s = $M_s r/T_{static}$

= 101 g x 48"/.63 mils p-p

= 7695 g-in/mil p-p

S_s = 271 oz-in/mil p-p

Dynamic Sensitivity, S_d = $M_d r/T_{dynamic}$

= 123 g x 48"/1.28 mils p-p

= 4613 g-in/mils p-p

S_d = 163 oz-in/mils p-p

EXAMPLE 3: ROTOR MODEL

Here is an example of a non-symmetrical rotor with a very flexible shaft. The layout is shown in Figure 10.

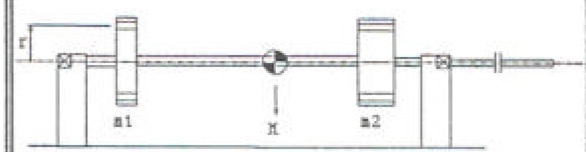

FIGURE 10 Rotor Model

$M = 1612$ g
$m_1 = 629$ g
$m_2 = 816$ g

$w = 5000$ rpm
$w_c = 3500$ rpm
$r = 1.2"$
$c = .66$

Estimated and actual sensitivities are as follows:

Static Sensitivity:

Estimated, S_s = .0284 oz-in/mil p-p
Actual, S_s = .0177 oz-in/mil p-p

Dynamic Sensitivity:

Estimated, S_D = .0102 oz-in/mil p-p
Actual, S_D = .0289 oz-in/mil p-p

The discrepancies between actual and estimated values result from the fact that the shaft is neither rigid or symmetrical.

It should be noted that there are additional sources of dynamic unbalance when the balance weights at a plane are not symmetrically applied.

Table 1 summarizes the estimated and actual sensitivities of the three examples presented.

TABLE 1 STATIC AND DYNAMIC BALANCE SENSITIVITIES – ESTIMATED VS. ACTUAL

CASE STUDY	W/w c1 / W/w c2	STATIC SENSITIVITY			DYNAMIC SENSITIVITY		
		ACTUAL oz-in/mpp	ESTIMAT-ED oz-in/mpp	ACT/ EST	ACTUAL oz-in/mpp	ESTIMAT-ED oz-in/mpp	ACT/ EST
Rotor Model 1.6 kg Rotor 5000 rpm	1.43 .71	.0177	.0284	.62	.0289	.0102	2.83
Generator 11,800 kg 3600 rpm	2.67 <1	200	208	.96	14-21	21.8	.61 - .96
Dryer Exhaust Fan 10,000 kg 1190 rpm	1.35 <1	271	180	1.5	163	224	.73

This type of modal balancing enables a good first estimate of balance sensitivities when the rotor is rigid and symmetrical about its center of gravity. The degree to which the rotor becomes flexible and non-symmetrical determines the variation between the estimated and actual sensitivities.

References: Csokmay, J.M.,
"Rotor-Balancing by Static-Couple Derivation."
IRD Mechanalysis, Inc.
Columbus, Ohio